A New Generalization of the (2+1)-dimensional Davey-Stewartson Equation

Ji Lin[a,b,c], Xiao-yan Tang[a], Sen-yue Lou[a], and Ke-lin Wang[b]

[a] Physics Department of Shanghai Jiao Tong University, Shanghai 200030, China
[b] Center of Nonlinear Science, University of Science and Technology of China, Hefei 230026, China
[c] Department of Physics, Zhejiang Normal University, Jinghua 321004, China

Reprint requests to Prof. S.-y. L. E-mail: sylou@mail.sjtu.edu.cn

Z. Naturforsch. **56 a,** 613–618 (2001); received June 28, 2001

Using an asymptotically exact reduction method based on Fourier expansion and spatiotemporal re-scaling, a new integrable system of the nonlinear partial differential equation in (2+1)-dimensions, extended Davey-Stewartson I equation, is deduced from a known (2+1)-dimensional integrable equation. The integrability of the new equation system is explicitly proved by the spectral transformation. Actually, the corresponding Lax pair of the new equations can be obtained by applying the same reduction method to the Lax pair of the original equation.

Key words: Integrable Models; Davey-Stewartson I Equation; Fourier Asymptotical Expansion.

1. Introduction

Finding new integrable models in (1+1)- or in higher dimensions is one of the important topics in the nonlinear science. Various powerful methods have been developed to deal with this problem. For instance, many finite dimensional Hamiltonian systems (ordinary differential equation (ODE) systems) can be obtained from symmetry constraints of the (1+1)-dimensional integrable partial differential equations (PDEs) [1 - 6]. Some types of known (1+1)-dimensional integrable models can be similarly considered as the constraints of some (2+1)-dimensional integrable equations [7, 8]. Some special types of higher dimensional integrable models can be found by using the (1+1)-dimensional strong symmetries [9], the conformal invariance of the Schwartz form of the known lower dimensional integrable equations[10], the general Virasoro symmetry algebras [11] and the non-invertible deformation theory [12], etc.

Many integrable systems which have the same dimensions as those of the original models can be obtained by using the asymptotically exact reduction method (AERM) based on Fourier expansion from some known integrable equations (such as the Kadomtsev-Petviashivilli (KP), (2+1)-dimensional Korteweg de Vries (KdV) and the Nizhinik-Novikov-Veselov (NNV) equations). It is clear that, from the AERM, the integrability is inherited through a limit technique [13, 14]. Thus the AERM provides a powerful tool not only to investigate the integrability of known equations but also to derive new integrable models likely to be relevant in applicative contexts. The method can also be used to construct approximate solutions for weakly nonlinear ordinary differential equations [15].

In this paper, we would like to apply the AERM proposed by Calogero and Maccari [13], to a (2+1)-dimensional integrable system which is solvable by the inverse scattering method. A new (2+1)-dimensional system, extended Davey-Stewartson I equation, which is integrable under the same meaning can be obtained.

The basic idea of the AERM is to consider the nonlinear evolution PDE in the following way: Firstly, one linearizes the nonlinear PDE to obtain a linearized equation, i. e., the equation obtained after neglecting all the nonlinear terms of the nonlinear PDE. As we know, the linear equation can be best described in term of Fourier modes which have a constant amplitude and a well-defined group velocity $\boldsymbol{V} = (V_1(K_1, K_2), V_2(K_1, K_2))$ depending on the wave numbers K_1, K_2. The group velocity represents that a wave packet peaked of Fourier mode would

0932–0784 / 01 / 0900–0613 $ 06.00 © Verlag der Zeitschrift für Naturforschung, Tübingen · www.znaturforsch.com

move in configuration space. Secondly, to evaluate the weak nonlinear effects, we consider the specific Fourier mode and introduce the following frame of reference via the transformation:

$$\xi = \epsilon^p(x - V_1 t),\ \eta = \epsilon^p(y - V_2 t),\ \tau = \epsilon^q t; \quad (1)$$

$p, q \geq 0$, and ϵ is the sufficiently small expansion parameter. The modulation of the amplitude of that Fourier mode (that would remain constant in the linearized equation) is taken place. The amplitude of the Fourier mode is dependent on space and time, and then on the re-scaled variables ξ, η, τ that account for the need to look on larger space and time scales and to derive a non-negligible effect from the nonlinear terms.

Generally, as pointed out by Calogero et al. [16], the AERM can be used to any nonlinear partial differential equations (PDEs), no matter whether they are integrable or not. In Calogero's philosophy, the method will preserve the integrability, i. e., if the original model is integrable then the reductions of the model will be integrable also. In (1+1)-dimensions, the models obtained in this way are always of Nonlinear Schrödinger type [16] while in (2+1)-dimensions the models obtained from the AERM are of Davey-Stewartson (DS) type [14, 15]. More details on the method and the generic properties of PDEs for which the method seems applicable can be found in [16] for (1+1)-dimensional PDEs and in [14, 15] for (2+1)-dimensional PDEs.

2. A New Nonlinear PDE from a (2+1)-integrable System

We consider the (2+1)-dimensional system that is integrable by means of the spectral transform

$$U_t - U_{xxx} - 6\beta U U_x - \frac{2}{3}\alpha^2\beta U^2 U_x + 3W_y + 3\alpha U_x W = 0, \quad (2)$$

$$W_x = U_y, \quad (3)$$

where $U = U(x,y,t)$, $W = W(x,y,t)$, the subscripts represent differentiations with respect to variables x, y, t, and α, β are some real parameters. Equations (2) with (3) may be called generalized modified Kadomtsev-Petviashivilli equation (GMKPE). The linear dispersive part (2) can be described by the Fourier modes, while the dispersion relation and the group velocity read

$$\omega = K_1^3 + \frac{3K_2^2}{K_1}, \quad (4)$$

$$V_1 = 3K_1^2 - \frac{3K_2^2}{K_1^2},\ V_2 = 6\frac{K_2}{K_1}. \quad (5)$$

The envelope of a wave packet concentrated around a value $\boldsymbol{K}$ travels with constant group velocity (5) and slowly disperses. After neglecting the last term of (2), the model can be derived in many real physics fields like plasma physics [17], fluid physics [18], solid state physics [19], lattice dynamics [20] and physics of ferromagnetic materials [21]. Usually, to consider the weak nonlinear effects, it is sufficient to consider the term $\beta U U_x$. However, in some special types of physics problems, the coefficient of the lowest order nonlinearity is so small that the next order of nonlinearities like $U^2 U_x$ and $U_x W$ has to be included. The term $U^2 U_x$ has been really derived from many physics fields [17 - 21] while the introduction of the term $U_x W$ suggested by Konopelchenko and Dubrovsky is to guarantee the integrability of the model.

We use a formal asymptotic Fourier expansion

$$U(x,y,t) = \sum_{n=-\infty}^{n=+\infty} \epsilon^{\gamma_n} \psi_n(\xi,\eta,\tau;\epsilon) \cdot \exp[in(K_1 x + K_2 y - \omega t)], \quad (6)$$

where $\gamma_n = \sqrt{n}$ when $n \neq 0$, $\gamma_0 = r$ is a rational number to be determined later and $\psi_n(\xi,\eta,\tau;\epsilon) = \psi_n^*(\xi,\eta,\tau;\epsilon)$, because $U(x,y,t)$ is real. The limit of $\psi_n(\xi,\eta,\tau;\epsilon)$ for $\epsilon \rightarrow 0$ exists and is finite. The analogue of $W(x,y,t)$ is

$$W(x,y,t) = \sum_{n=-\infty}^{n=+\infty} \epsilon^{\gamma_n} \phi_n(\xi,\eta,\tau;\epsilon) \cdot \exp[in(K_1 x + K_2 y - \omega t)]. \quad (7)$$

It is clear in the Fourier expansion (6) that $\psi_n(\xi,\eta,\tau;\epsilon)$ depend on the parameter ϵ. Moreover, $\psi_n(\xi,\eta,\tau;\epsilon)$ can be expanded in a power series of ϵ, i. e.,

$$\psi_n(\xi,\eta,\tau;\epsilon) = \sum_{i=0}^{\infty} \epsilon^{(i)} \psi_n^i(\xi,\eta,\tau). \quad (8)$$

For simplicity of the following calculation, we introduce the abbreviations $\psi_n^{(0)} = \psi_n$ (for $n \neq 0, 1$), $\psi_1^{(0)} = \Psi$, $\psi_0^{(0)} = \Psi_0$, (and $\phi_n^{(0)} = \phi_n$, (for $n \neq 0, 1$) $\phi_1^{(0)} = \phi$, $\phi_0^{(0)} = \Phi$).

The final goal is to get the evolution equation of the modulation amplitude $\Psi = \Psi(\xi, \eta, \tau)$ and to know how it is to be modified by selecting different wave numbers.

The standard procedure of the reduction method is to consider the different equations obtained from the coefficients of the Fourier modes $\exp(in(K_1 x + K_2 y - \omega t))$. Substituting (6) and (7) into (2) and (3), we obtain

$$\phi = \frac{K_2}{K_1}\Psi,\ \phi_n^{(p)} = \frac{i}{nK_1}\left(\frac{K_2}{K_1}\psi_{n\xi}^{(0)} - \psi_{n\eta}^{(0)}\right), \quad (9)$$

$$\phi_n^{(2p)} = -\frac{1}{n^2 K_1^2}\left(\frac{K_2}{K_1}\psi_{n\xi\xi}^{(0)} - \psi_{n\xi\eta}^{(0)}\right), \Phi_\xi = \Psi_{0,\eta}. \quad (10)$$

For (2), it is more convenient to separate the contributions from the linear and nonlinear parts. Then (2) can be rewritten as

$$\epsilon^{\gamma_n} D_n \psi_n = \epsilon^2 F_n, \quad (11)$$

where D_n is a linear differential operator acting on $\psi_n(\xi, \eta, \tau)$, the corresponding expression of D_n is

$$D_n = -in\omega + \epsilon^q \partial_\tau - V_1 \epsilon^p \partial_\xi - V_2 \epsilon^p \partial_\eta - (inK_1 + \epsilon^p \partial_\xi)^3 + 3(inK_2 + \epsilon^p \partial_\eta)\Big(\frac{K_2}{K_1} + \epsilon^p \frac{i}{nK_1}\Big(\frac{K_2}{K_1}\partial_\xi - \partial_\eta\Big) + \frac{\epsilon^{2p}}{n^2 K_1^2}\Big(\partial_{\xi\eta} - \frac{K_2}{K_1}\partial_{\xi\xi}\Big)\Big) + o(\epsilon^{3p}), \quad (12)$$

and F_n represent the contribution of the nonlinear part. It is easy to find that F_n ($n = 0, 1, 2$) have the following explicit expressions

$$F_0 = 6\Big(\beta - \alpha\frac{K_2}{K_2}\Big)(|\Psi|^2)_\xi \epsilon^p + 3\alpha(|\Psi|^2)_\eta \epsilon^p + o(\epsilon^{p+2}), \quad (13)$$

$$F_1 = 6\beta(iK_1 \Psi^* \psi_2 \epsilon + iK_1 \Psi_0 \Psi \epsilon^{r-1}) + \frac{2}{3} i\alpha^2 K_1 |\Psi|^2 \Psi \epsilon - 3\alpha(iK_2 \Psi^* \psi_2 \epsilon + iK_1 \Phi \Psi \epsilon^{r-1} + o(\epsilon^{p+r-1}, \epsilon^{2p}), \quad (14)$$

$$F_2 = 3i(2\beta K_1 - \alpha K_2)\Psi^2 + o(\epsilon^p). \quad (15)$$

In order to make the proper balance of terms, we choose $p = 1, q = 2, r = 2$ and then calculate for $n = 0, n = 1, n = 2$. The evolution equations about the functions Ψ, Φ, Ψ_0 satisfy

$$-V_1 \Psi_{0,\xi} - V_2 \Psi_{0,\eta} + 3\Phi_\eta + 3\alpha|\Psi|_\eta^2 + 6\Big(\beta - \alpha\frac{K_2}{k_1}\Big)|\Psi|_\xi^2 = 0 \quad (16)$$

$$i\Psi_\tau + \Big(2K_1 + 3\frac{K_2^2}{K_1^3}\Big)\Psi_{\xi\xi} - 6\frac{K_2}{K_1^2}\Psi_{\xi\eta} + \frac{3}{K_1}\Psi_{\eta\eta} + \chi\Psi - 3\alpha K_1 \Phi\Psi = 0 \quad (17)$$

with

$$N = \frac{2}{3}\alpha^2 K_1 + \frac{2}{3}\alpha^2\frac{K_2^2}{K_1^3} + 6\frac{\beta^2}{K_1} - 6\alpha\beta\frac{K_2}{K_1^2},$$

$$\chi = 6\beta K_1 \Psi_0 + N|\Psi|^2, \quad (18)$$

and

$$\psi_2^{(0)} = \Big(\frac{\beta}{K_1^2} - \frac{\alpha K_2}{2K_1^3}\Big)\Psi^2. \quad (19)$$

For the simplicity of (16) and (19), eliminating Ψ_0 from (18), we have

$$\chi_\eta = 6\beta K_1 \Phi_\xi + N|\Psi|_\eta^2, \quad (20)$$

$$V_1 \chi_{\xi\xi} + V_2 \chi_{\xi\eta} - 3\chi_{\eta\eta} = (-36K_1\beta^2 + 36\alpha\beta K_2 + NV_1)|\Psi|_{\xi\xi}^2 + (NV_1 - 18\alpha\beta K_1)|\Psi|_{\xi\eta}^2 - 3N|\Psi|_{\eta\eta}^2 \quad (21)$$

Thus, (17), (20), and (21) are a (2+1)-dimensional system for three counterparts Φ, χ, and Ψ. It is interesting that, if we take $\alpha = 0, \beta \neq 0$ or $\alpha \neq 0, \beta = 0$ in (17), (20) and (21), and make trivial re-scaling, we get the Davey-Stewartson I equation that has wide applicative relevance in plasma physics, nonlinear optics, etc. It is known that the DSI equation possesses dromion like solutions which decay exponentially in all directions [22, 23]. Another type of DS equation (DS II equation) had been extended by Maccari in 1999 [14]. The DS II equation possesses only lump type localised solutions which decay algebraically in all directions [24]. Moreover, we can obtain the (1+1)-dimensional NLS equation if $\xi = \eta$.

Comparing (17), (20) and (21) with the standard DS I system we see that the essential difference between the extended DSI and the standard DSI models is the entering of the second mean flow field Φ caused by the higher order nonlinearities ($\alpha \neq 0$). In other words, the entering of the second order nonlinearities will offer an additional mean motion for the solitary waves of the DS system which may describe the solitary waves in many real physics systems like plasma physics [17], fluid physics [18], solid state physics [19], lattice dynamics [20] and physics of ferromagnetic materials [21]. In general the nonlinear term $\beta U U_x$ is much larger than the higher term $\alpha U^2 U_x$, and the mean motion is mainly caused by the filed χ. However sometimes β may be very small. In that case the higher order nonlinearity terms have to be included in the mean motion.

For $\alpha \neq 0$ and $\beta \neq 0$ in (17), (20) and (21), we take the following re-scalings:

$$\xi' = -\frac{\alpha}{2\sqrt{3}\beta}\xi,\ \eta' = \frac{1}{\sqrt{3}}\eta,\ \tau' = \frac{\tau}{K_1},$$

$$\Psi' = \sqrt{|N|K_1}\Psi,\ \chi' = K_1\chi,\ \Phi' = -3\alpha K_1^2\Phi, \quad (22)$$

and derive the simpler form of the new system

$$i\Psi_\tau + L_1\Psi + \chi\Psi + \Phi\Psi = 0,$$

$$L_2\chi = L_3|\Psi|^2,$$

$$\Phi_\xi = \chi_\eta \pm |\Psi|^2_\eta, \quad (23)$$

where

$$L_1 = \frac{a^2+b^2}{4}\frac{\partial^2}{\partial\xi^2} + a\frac{\partial^2}{\partial\xi\partial\eta} + \frac{\partial^2}{\partial\eta^2},$$

$$L_2 = \frac{b^2-a^2}{4}\frac{\partial^2}{\partial\xi^2} - a\frac{\partial^2}{\partial\xi\partial\eta} - \frac{\partial^2}{\partial\eta^2},$$

$$L_3 = \pm\Big(\frac{b^2-a^2}{4} + \frac{2(a-1)b^2}{(a-2)^2+b^2}\Big)\frac{\partial^2}{\partial\xi^2}$$

$$\mp\Big(a - \frac{2b^2}{(a-2)^2+b^2}\Big)\frac{\partial^2}{\partial\xi\partial\eta} \mp \frac{\partial^2}{\partial\eta^2}$$

with

$$a = \frac{\alpha K_2}{\beta K_1},\ b = \frac{\alpha K_1}{\beta}.$$

$\Psi = \Psi(\xi,\eta,\tau)$ is complex, $\Phi = \Phi(\xi,\eta,\tau)$ and $\chi = \chi(\xi,\eta,\tau)$ are real.

The equations (23) obtained from an (2+1) integrable equation by the inverse scattering method is the general DSI equation. In the next section, the integrability of the new system will be identified.

3. The Lax Pair of the New System

We can find that the Lax pair of the original equation has the form

$$L = i\partial_y + \partial_x^2 - i\alpha u\partial_x + \beta u,\ L\varphi(x,y,t) = 0, \quad (24)$$

$$A = -4\partial_x^3 + 6i\alpha u\partial_x^2 - 3\beta u_x + \frac{2}{3}i\alpha\beta u^2 + 3i\beta w$$

$$+ 3i\alpha u_x\partial_x + \frac{2}{3}\alpha^2 u^2\partial_x - 6\beta u\partial_x \quad (25)$$

$$+ 3\alpha w\partial_x \varphi_t(x,y,t) + A\varphi(x,y,t) = 0.$$

In order to demonstrate that the new system is integrable by means of the spectral transform, we also apply the reduction method to the Lax pair (24) and (25), identifying the Lax pair that permits one to get a compatibility condition which reproduces the Equations (23).

Similarly, we can introduce the asymptotic Fourier expansion of φ

$$\varphi(x,y,t) = \sum_{n=-\infty}^{n=+\infty} \epsilon^{\gamma_n}\varphi_n(\xi,\eta,\tau;\epsilon) \quad (26)$$

$$\cdot \exp\Big(i\frac{n}{2}z + i(\lambda_1 x + \lambda_2 y + \lambda_3 t)\Big),$$

where n is odd, $z = K_1 x + K_2 y - \omega t$, $\gamma_{n+2} = 1 + \gamma_n$, $\gamma_{-n} = \Gamma_n$ (when $n > 0$), $\lambda_1, \lambda_2, \lambda_3$ are constants to be determined later on, and $\varphi_n(\xi,\eta,\tau,\epsilon)$ is dependent on ϵ.

Inserting the expression of $\varphi(x,y,t)$ into (24) and (25), we obtain a series of relations which are generated by the coefficients of the Fourier modes. Obviously, each relation should be valid for a given order of approximation in ϵ. In particular, for the fundamental harmonics $\varphi_\pm(x,y,t) \equiv \varphi_{\pm 1}(x,y,t)$ in ϕ, from terms $o(\epsilon^0)$ in (24) and (25), we can determine the constants $\lambda_1, \lambda_2, \lambda_3$:

$$\lambda_1 = -\frac{K_2}{2K_1},\ \lambda_2 = -\frac{K_1^2}{4} - \frac{K_2^2}{4K_1^2},$$

$$\lambda_3 = \frac{2}{3}K_1K_2 + \frac{K_2^3}{2K_1^3}. \tag{27}$$

The new spectral problem is given from the successive order ϵ in Equations (24).

$$i\varphi_{+,\eta} + i\Big(K_1 - \frac{K_2}{K_1}\Big)\varphi_{+,\xi} + \Big(\beta - \alpha\Big(\frac{K_2}{2K_1} + \frac{K_1}{2}\Big)\Big)\Psi\varphi_- = 0$$

$$i\varphi_{-,\eta} - i\Big(K_1 + \frac{K_2}{K_1}\Big)\varphi_{-,\xi} + \Big(\beta + \alpha\Big(-\frac{K_2}{2K_1} + \frac{K_1}{2}\Big)\Big)\Psi^*\varphi_+ = 0. \tag{28}$$

We obtain the above equations from the same order ϵ in (25). In order to get other equations of the Lax pair of the new (2+1)-dimensional system (17), (20) and (21), we must take into account the next order (ϵ^2) of approximation of (25). The appearing new quantities, the corrections $\widetilde{\varphi}_\pm(\xi,\eta,\tau)$ of orderϵ to the fundamental harmonics $\varphi_\pm(\xi,\eta,\tau)$, can be eliminated in (25) by taking advantage of the relation obtained from (24) in the terms of the order ϵ^2. After a tedious calculation, finally, we can obtain the following Lax pair

$$L\hat{\varphi} = 0,\ \hat{\varphi}_\tau + A\hat{\varphi} = 0 \tag{29}$$

$$L = \begin{pmatrix} \partial_\eta + \frac{a-b}{2}\partial_\xi & i\frac{a+b-2}{\sqrt{2(a-2)^2+2b^2}}\Psi \\ i\frac{a-b-2}{\sqrt{2(a-2)^2+2b^2}}\Psi^* & \partial_\eta - \frac{a+b}{2}\partial_\xi \end{pmatrix},$$

$$\hat{\varphi} = \begin{pmatrix} \varphi_+ \\ \varphi_- \end{pmatrix} \tag{30}$$

and

$$A = \tag{31}$$

$$\begin{pmatrix} a_1\partial_\xi^2 + a_2\chi + a_3|\Psi|^2 + a_4\Phi & a_5\Psi\partial_\xi + a_6\Psi_\xi + a_7\Psi_\eta \\ b_4\Psi^*\partial_\xi + b_6\Psi_\xi^* + b_7\Psi_\eta^* & b_1\partial_\xi^2 + b_2\chi + b_3|\Psi|^2 + b_4\Phi \end{pmatrix},$$

where

$$a_1 = \frac{ib^2}{2},\ a_2 = i\frac{a(a+b)(a-b-2)}{4b},$$

$$a_3 = -i\frac{((a-2)^2 - b^2)(a^3 - 2a^2 - 2ab + ab^2 - 2b^2)}{4b^2((a-2)^2 + b^2)},$$

$$a_4 = i\frac{a(a-b-2)}{2b^2},\ a_5 = \frac{(a+b)(a+b-2)}{2\sqrt{2(a-2)^2 + 2b^2}},$$

$$a_6 = -\frac{a(a+b-2)}{\sqrt{2(a-2)^2 + 2b^2}},\ a_7 = \frac{a+b-2}{\sqrt{2(a-2)^2 + 2b^2}},$$

$$b_1 = \frac{-i}{2}(2a^2 + 2ab + b^2),\ b_2 = -i + a_2,\ b_3 = a_3,$$

$$b_4 = a_4 - i,\ b_5 = \frac{(3a+b)(b+2-a)}{2\sqrt{2(a-2)^2 + 2b^2}},$$

$$b_6 = \frac{a(b+2-a)}{2\sqrt{2(a-2)^2 + 2b^2}},\ b_7 = \frac{2+b-a}{\sqrt{2(a-2)^2 + 2b^2}}.$$

The general DSI equation can be obtained from the consistent condition of the Lax pair

$$L_\tau = [L, A]. \tag{32}$$

4. Summary and Discussion

By means of the AERM based on the Fourier expansion and spatiotemporal re-scaling, we obtain a new generalization of the (2+1)-dimensional DSI system from a (2+1)-dimensional general modified Kadomtsev-Petviashvili type integrable equation. Comparing the extended DSI equation with the standard DSI equation, an additional mean flow field is introduced because of the higher order nonlinearity of the original model. It is found that the new DSI system is integrable under the meaning that it possesses Lax pair and then can be solved by means of the inverse scattering transformation approach.

The results of this paper show that the AERM based on Fourier expansion proposed by Calogero and Maccari is a powerful method to derive some new integrable models from known ones. Mathematically, every integrable model can be used as a source to obtain new integrable models by means of several reduction methods including the AERM. In principle one may use one method, say AERM, to obtain many integrable models recursively from a known single one. In some special cases, the recursive will be ended, that means if one uses the obtained new model as a source, one may obtain only the original equation from [25]. However, in real physics application, it is not useful to use the obtained model as a new source to look for further approximate integrable models because the valid conditions of the approximations may be quite different. For instance, the valid condition to obtain the generalized DSI equation, one has used the "short wave" (envelope wave) approximation. However if we use the obtained DSI equation to obtain the GMKP equation one has to use the "long wave" approximation.

Calogero, Degasperis and Ji [16] had pointed out that in the (1+1)-dimensional case, the AERM always lead to the NLS type equations. A similar situation occurs in the (2+1)-dimensional case, the AERM always leads to the DS type equations which describes effects of the weak nonlinearity on the linear Fourier modes. Though the set of solutions obtained by the AERM can be used to describe the nonlinear effects on the linear Fourier modes well, many real physical solutions beyond the "short wave" approximation and the weak nonlinearity of the originall model will be lost.

Moreover, we believe that this method may be used also in higher dimensions. In future studies, we hope to obtain some (3+1)-dimensional "more" integrable models from (3+1)-dimensional Painlevé (or Virasoro) integrable models [10] by using the AERM based on the Fourier expansion.

This work was supported by the Outstanding Youth Foundation and the National Natural Science Foundation of China (Grant. No. 19925522), the Research Fund for the Doctoral Program of Higher Education of China (Grant. No. 2000024832) and the Natural Science Foundation of Zhejiang Province, China. The authors are in debt to thank the referee for his comments and the helpful discussions with the professors Q. P. Liu and G.-x. Huang.

[1] C. S. Gardner, J. M. Green, M. D. Kruskal, and R. M. Miura, Comm. Pure. Appl. Math. **27**, 97 (1974).
[2] B. G. Konopenlcnenko, J. Sidorenko, and W. Strampp Flaschka, Nonlinear Integrable Systems-Classical and Quantum Theory, Edts. M. Sato, M. Jimbo, and T. Miwa, World Scientific, Singapore 1983, p. 39.
[3] C. W. Cao, Henan Sci. **5**, 1 (1987); Chin. Quart. J. Math. **3**, 90 (1988); Acta Math. Sinica, New Series **6** (1990); Sci. China **A33**, 528 (1990).
[4] C. W. Cao and X. G. Geng, J. Phys. A: Math. Gen. **23**, 4117 (1990); J. Math. Phys. **23**, 2323 (1991).
[5] Y. B. Zeng and Y. S. Li, J. Math. Phys. **30**, 1617 (1989); J. Phys. A: Math. Gen. **23**, L89 (1990); Y. B. Zeng, Phys. Lett. **A160** (1991) 541.
[6] G. Z. Tu, J. Math. Phys. **30**, 330 (1989).
[7] B. Konopenlcnenko, J. Sidorenko, and W. Strampp, Phys. Lett. **A157**, 17 (1991).
[8] Y. Cheng and Y. S. Li, Phys. Lett. **A157**, 22 (1991); J. Phys. A: Math. Gen. **25**, 419 (1992).
[9] S.-y. Lou, Sci. China **A40**, 1317 (1997).
[10] S.-y. Lou, Phys. Rev. Lett. **80**, 5027 (1998); J. Math. Phys. **39**, 2121 (1998); S.-y. Lou and J.-j. Xu, J. Math. Phys. **80**, 5364 (1998).
[11] J. Lin, Commun. Theor. Phys. **25**, 447 (1996); S.-y. Lou, J. Lin, and J. Yu, Phys. Lett. **A201**, 47 (1995); J. Lin, S.-y. Lou, and K.-l. Wang, Z. Naturforsch. **55a**, 589 (2000); J. Lin and K.-l. Wang, Atca Phys. Sinica **50**, 11 (2001).
[12] J. Yu and S.-y. Lou, Sci. China **A43**, 655 (2000).
[13] F. Calogero and W. Eckhaus, Inv. Probl. **3**, 229 (1988); **4**, 11 (1988).
[14] A. Maccari, J. Math. Phys. **37**, 6207 (1996); **38**, 4151 (1997); **40**, 3971 (1999); **42**, 2689 (2001).
[15] A. Maccari, Int. J. Nonlinear Mech. **33**, 713 (1998); Nonlinear Dynamics **15**, 329 (1998).
[16] F. Calogero, A. Degasperis, and X.-d. Ji, J. Math. Phys. **41**, 6399 (2000); **42**, 2635 (2001).
[17] I. S. Okeir and E. J. Parkes, Physica Scripta **55**, 135 (1997).
[18] H. Tsuji and M. Oikawa, J. Phys. Soc. Japan **62**, 3881 (1993).
[19] S. Erbay, Int. J. Nonlinear Mach. **34**, 289 (1999); C. Babaolu and S. Erbay, Chaos, Solitons and Fractals **12**, 381 (2001).
[20] K. S. Sreelatha and K. B. Joseph, Chaos, Solitons and Fractals **11**, 711 (2000).
[21] N. Papanicolaou and P. N. Spathis, Nonlinearity **12**, 285 (1999).
[22] A. S. Fokas and P. M. Santini, Phys. Rev. Lett. **63**, 1329 (1989); Physica D **44**, 99 (1990); M. Boiti, J. J. P. Leon, L. Martina, and F. Penpinelli, Phys. Lett. **132A**, 432 (1988).
[23] S.-y. Lou and J.-z. Lu, J. Phys. A. **29**, 4209 (1996).
[24] M. Manas and P. M. Santini, Phys. Lett. A **227**, 325 (1997).
[25] V. E. Zakharov and E. A. Kuznetsov, Physica D **18**, 455 (1986).